Algebrinis Bethe anzatsas integruojamiems vienmačiams magnetams

Algebraic Bethe ansatz for integrable one-dimensional magnets

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The Bethe ansatz is a large collection of methods to find the spectrum and common eigenvectors of commuting families of operators (transfer matrices) occurring in the theory quantum integrable one-dimensional models. It was the Faddeev's school of mathematical physics which reformulated the spectral problem of quantum integrable models into a question of representation theory of certain associative algebras with quadratic relations, now generally known as quantum groups. More precisely, the spaces of states of such models, called quantum spaces, are associated to tensor products of irreducible representations of these quantum groups. The commuting operators are then images of elements in the commutative subalgebra, known as Bethe subalgebra, on the quantum space. By acting with appropriate algebra elements on the vacuum vectors one then constructs the so-called Bethe vectors that depend on sets of complex parameters. In the case when these parameters satisfy certain algebraic equations, known as Bethe equations, the corresponding Bethe vectors become eigenvectors of the commuting operators. In this form, the Bethe ansatz is called algebraic Bethe ansatz. The general conjecture is that the constructed eigenvectors form a basis in the space of states of the model.

Algebraic Bethe ansatz has been very fruitful in the study of the gl_N-symmetric integrable models, the most famous case being the Heisenberg spin chain describing the spectrum of spin-waves, critical points and phase transitions in one-dimensional (anti-)ferromagnets. The study of the so_N- and sp_N-symmetric models so far has been less productive. Such models were first studied using analytic Bethe ansatz techniques in [1]. One of the obstacles to the algebraic approach is that the scattering matrix in this case is not quite of a six-vertex type, which is the key property used in the study of the gl_N-symmetric models. Another obstacle is that not every irreducible highest weight so_N- or sp_N-representation can be lifted to a representation of the corresponding quantum group, such as Yangian or quantum loop algebra. Moreover, the lifting itself is often not straightforward and requires a usage of the fusion procedure or some other method, such as spinor or oscillator algebra realization. Consequently, the algebraic study of one-dimensional so_N- or sp_N-symmetric spin chains has mostly been restricted to the cases, when the quantum space of the model is a tensor product of fundamental representations (fundamental models).

In this talk I will give a pedestrian introduction to the algebraic Bethe ansatz for the Heisenberg spin chain and explain difficulties that arise when transitioning from gl_N-symmetric to so_N- and sp_N-symmetric quantum integrable one-dimensional magnets. The talk will be based on my joint work with Allan Gerrard [2]. In this work we study the spectral problem of so_N- and sp_N-symmetric open spin chains with more general quantum spaces and certain diagonal open boundary conditions. The algebraic structure of such models is described by the extended twisted Yangians introduced by *N.* Guay and myself in [3]. We focus on the $N = 2n$ case. The decomposition $\mathbb{C}^{2n} \cong \mathbb{C}^2 \otimes \mathbb{C}^n$ then allows us to rewrite the scattering matrix of the chain as an $\mathrm{End}(\mathbb{C}^n \otimes \mathbb{C}^n)$-valued six-vertex matrix and thus apply the conventional algebraic Bethe ansatz methods, subject to necessary modifications, to solve the spectral problem of the chain. The space of states is given by a tensor product of symmetric irreducible so_{2n}-representations or by a tensor product of skew-symmetric irreducible sp_{2n}-representations. We obtain explicit formulae of Bethe vectors, their eigenvalues and algebraic Bethe equations. This provides the necessary first step that needs to be taken in the study of physical properties such as scalar products and norms, correlation functions and form factors for these one-dimensional magnets. The study of these models may also provide insights to stochastic systems such as Markov chains, reaction-diffusion models and certain biological transport systems [4]. Another important aspect of this study is a link to the four-dimensional Gauge theories and three-dimensional Chern-Simons theories via the Cotello-Witten-Yamazaki theory recently discovered in [5].

Keywords: algebraic Bethe ansatz, integrable open spin chains, one-dimensional magnets, twisted Yangian.

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